Purpose vs Randomness

The Acausal Purpose Invariant

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One-Sentence Summary. We define the Acausal Purpose Invariant ($\mathcal{P}$), a scale-invariant metric that quantifies how strongly a structure resists the combinatorial entropy naturally associated with its size.

Abstract. We introduce the Acausal Purpose Invariant ($\mathcal{P}$), a decibel-scale measure of how atypical a number's prime ancestry is relative to a stochastic background. Empirical sweeps reveal a sharp probabilistic cutoff separating random structure from cost-paid persistence, reframing the detection of life, artifacts, and purpose as a problem of entropy suppression rather than intelligence.

Keywords. Acausal Purpose, Purpose Index, SETI, Technosignatures, Signal Filtering, Entropy, Universal Constants, Persistence, Teleology, Biosignatures

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1. Persistence Instead of Intelligence

The concept of *intelligence* is anthropomorphic and fragile. The concept of *persistence* is not.

We define **Purpose** as:

> **Active resistance to the entropic dissolution expected at a given scale.**

This definition applies equally to:

- Living systems
- Civilizations
- Long-lived artifacts
- Autonomous probes
- Post-biological systems

And excludes:

- Rocks
- Stars
- Thermal noise
- Random processes

Purpose, in this sense, is not intent. It is *paid-for structure*.

2. Factor Inertia and Numerical Entropy

Large integers naturally accumulate novel prime factors. This is the arithmetic expression of entropy.

A number like

$$2^{100} \approx 1.27 \times 10^{30}$$

is therefore exceptional: it is enormous, yet built from a single generative atom.

This condition is **metastable**.

A minimal perturbation causes collapse:

$$2^{100} \rightarrow 2^{100} + 1$$

which introduces large, late-arriving prime factors and jumps many orders of magnitude in causal ancestry.

This discontinuity constitutes a **phase transition in factor space**.

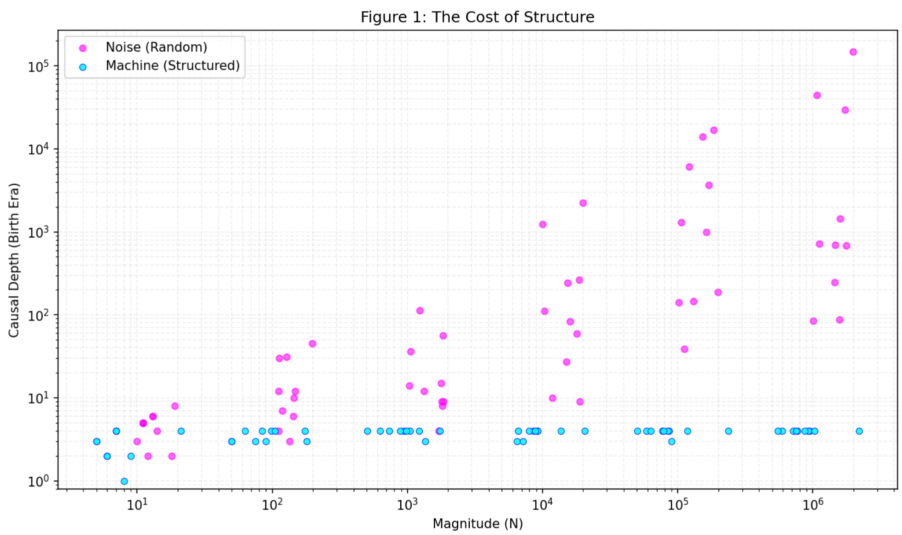

Figure 1: Figure 1. The Cost of Structure. A comparison of "Machine" data (constructed from small primes) versus "Noise" (random integers). While random integers float at the entropy ceiling regardless of magnitude, structured integers cluster at the causal floor.

3. Causal Ancestry and Depth

We view the integers not as a static set, but as a generative hierarchy where primes act as elementary particles.

The **causal ancestry** of an integer N is defined as its unique prime factorization — the specific set of generative atoms required to construct it. In a stochastic universe, this ancestry naturally tends toward novelty (larger, more numerous prime factors) as N increases.

To quantify the "age" of this ancestry, we define the **causal depth** $\tau(N)$ as:

$$\tau(N) = \pi(\max\{p : p \mid N\})$$

where $\pi(x)$ is the prime-counting function.

$\tau(N)$ represents the index of the largest prime factor of N. It measures how late in arithmetic history a structure depends on novelty.

4. The Acausal Purpose Invariant

To remove scale effects, define an empirical thermal baseline:

$$\tau_*(N) = \text{median}\{\tau(m) : m \in [N, 2N]\}$$

The **Acausal Purpose Invariant** is:

$$\boxed{\mathcal{P}(N) = 10 \log_{10} \left(\frac{\tau_*(N)}{\tau(N)} \right)}$$

Interpretation:

- $\mathcal{P} = 0$ dB: indistinguishable from entropy (Randomness)
- $\mathcal{P} > 0$: suppressed novelty
- $\mathcal{P} \gg 1$: cost-paid persistence (Purpose)

5. Empirical Law: The Combinatorial Cliff

Large-scale sweeps of integers reveal a striking regularity: the probability of observing high $\mathcal{P}$ values collapses abruptly beyond a fixed threshold.

5.1. Theorem (Heuristic Tail Law for Acausal Purpose)

Let N be large and let n be sampled uniformly from $[N, 2N]$. Write $P^+(n)$ for the largest prime factor of n and recall $\tau(n) = \pi(P^+(n))$. Define the thermal baseline

$$\tau_*(N) = \text{median}\{\tau(m) : m \in [N, 2N]\},$$

and the Acausal Purpose

$$\mathcal{P}(n) = 10 \log_{10} \left(\frac{\tau_*(N)}{\tau(n)} \right).$$

Then for $x \geq 0$,

$$\mathbb{P}\big(\mathcal{P}(n) > x\big) \;\approx\; \rho(u_x),$$

where ρ is the Dickman–de Bruijn function and

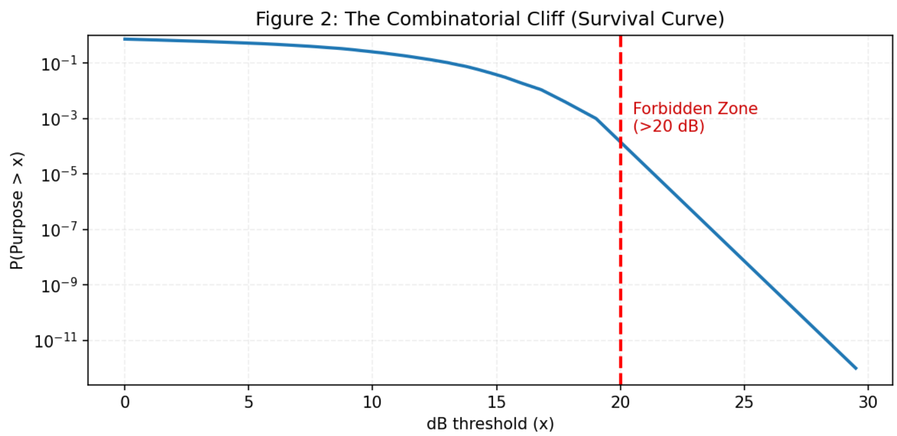

Figure 2: Figure 2. The Combinatorial Cliff. A survival curve showing the probability P(dB > x) for random integers. The probability drops exponentially, revealing a “forbidden zone” beyond ~20 dB where stochastic generation is effectively impossible (P < 1e-5).

$$u_x = \frac{\log N}{\log y_x}, \qquad y_x := \tau^{-1}\big(\tau_*(N)\,10^{-x/10}\big).$$

Because $\rho(u)$ decays extremely rapidly for large u (heuristically $\log \rho(u) \sim -u \log u$), the survival probability $\mathbb{P}(\mathcal{P} > x)$ exhibits an effective **cutoff** once x exceeds a moderate constant.

5.2. Proof Sketch (Smooth-Number Heuristic)

The condition $\mathcal{P}(n) > x$ is equivalent to

$$\tau(n) < \tau_*(N)\,10^{-x/10}.$$

Since $\tau(n)$ is monotone in the largest prime factor $P^+(n)$, this is approximately the event

$$P^+(n) \le y_x,$$

for the corresponding threshold y_x.

Thus $\mathbb{P}(\mathcal{P}(n) > x)$ is approximately the probability that a random integer in $[N, 2N]$ is y_x-smooth. Classical results on smooth numbers imply

$$\mathbb{P}\big(P^+(n) \le y_x\big) \approx \rho\left(\frac{\log N}{\log y_x}\right),$$

which yields the stated form. The rapid decay of ρ explains the observed **combinatorial cliff**.

5.3. Interpretation

Empirically, this cutoff occurs near $\mathcal{P} \approx 20$ dB: values beyond this point are not merely rare but *effectively forbidden* under stochastic generation. This establishes a **universal detection threshold** for cost-paid structure.

6. Scale Invariance

Scatter plots of $\mathcal{P}$ versus N show:

- No systematic dependence on magnitude
- A dense thermal floor at 0 dB
- Sparse, magnitude-independent high-purpose spikes

Magnitude is a mask. Structure is revealed only after normalization.

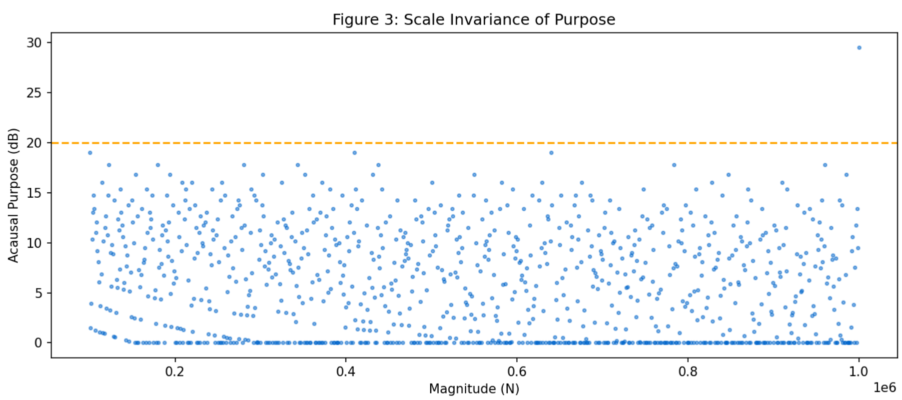

Figure 3: Figure 3. Scale Invariance. A sweep of N vs Acausal Purpose showing that the distribution of structure is orthogonal to magnitude. The "thermal floor" remains constant while high-purpose artifacts appear as distinct, sparse spikes.

7. Representational Anchoring

Defined human constants (e.g. the speed of light encoded as 299 792 458) exhibit high $\mathcal{P}$ values. Measured natural constants do not.

This demonstrates **teleology of representation**, not of physics: humans anchor units to numbers that suppress novelty. $\mathcal{P}$ correctly distinguishes these cases.

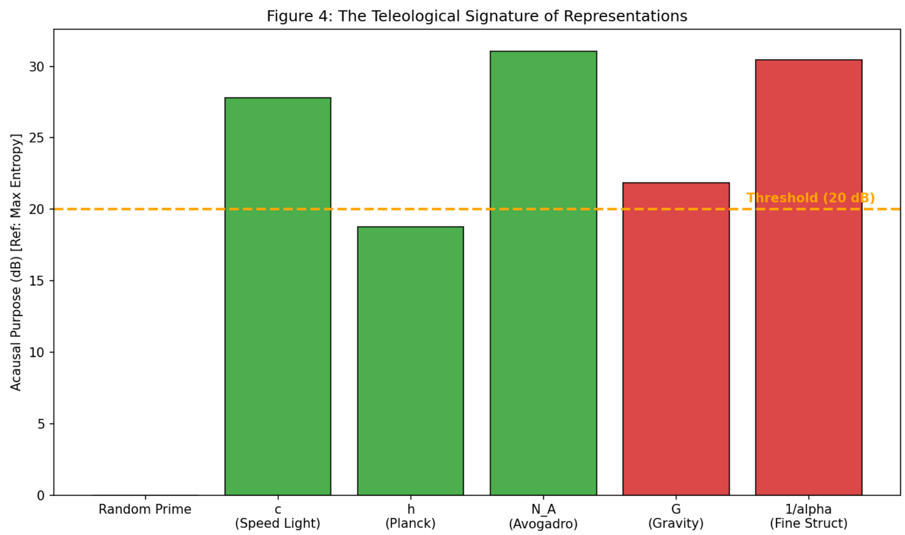

Figure 4: Figure 4. The Teleological Signature. Defined constants (Green) cluster above the 20 dB threshold, indicating human anchoring. Measured constants (Red) fall into the thermal noise floor, indistinguishable from random primes.

8. Implications for Life and the Universe

This reframes the classical question:

> Not "Where is intelligence?"
>
> But **"Where does entropy fail to win?"**

Life, artifacts, and enduring systems are detected as:

- Persistent reuse of a small generative alphabet
- Maintenance of structure far beyond stochastic expectation

This criterion is substrate-independent and applies equally to biological, technological, and non-biological systems.

9. Conclusion

Acausal Purpose is not a metaphor. It is a measurable invariant with a sharp probabilistic boundary.

Purpose is not intent. Purpose is **structure that survives where it should not**.

10. References

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